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DYNAMICS, HEAT AND MASS TRANSFER OF VAPOUR–GAS BUBBLES IN A LIQUID

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Abstract—A nonlinear problem of thermal, mass and dynamic interaction between a vapour–gas bubble and a liquid is considered with account for temperature nonuniformity in the bubble and interdiffusion of the vapour–gas mixture components. A numerical solution is obtained for the problem of radial bubble motion induced by a sudden pressure change in the liquid — a situation which, in particular, corresponds to the behaviour of bubbles beyond a shock-wave front when the latter enters a bubble curtain. Considered also are vapour–gas bubbles oscillating in the liquid under the influence of a sound field. The capillary effects and phase transitions, taken together, are shown to produce a new resonant frequency of small vapour bubbles which differs from that described by Minnaert. The expressions for the frequency and the thermal damping ratio of bubble oscillations are obtained. The effective coefficients of heat transfer between radially oscillating bubbles and the liquid are determined.

NOMENCLATURE

R, bubble radius;
$\dot{R}$, time derivative of the radius;
r, radial Euler coordinate;
t, time;
T, temperature;
ρ, density;
p, pressure;
k, concentration;
v, velocity;
w, diffusion rate;
u, specific internal energy;
λ, thermal conductivity;
D, interdiffusion coefficient;
B, gas constant;
j, rate of phase transitions from a unit surface;
κ, accommodation coefficient;
μ, viscosity;
l, specific heat of vaporization;
γ, specific heat ratio;
C, speed of sound in a gas;
c, specific heat;
a, thermal diffusivity;
f, frequency of oscillations;
ω, $= 2\pi f$, circular frequency;
m, mass of a bubble;
Λ, logarithmic decrement;
S, resonant function;
p_A, acoustic pressure amplitude;
α, nondimensional displacement of bubble surface;
β, phase of oscillations;
Ψ, bubble compressibility;
σ, surface tension coefficient.

Subscripts

l, liquid;
v, vapour;
g, gas;
s, at saturation;
σ, on bubble surface;
0, at equilibrium;
∞, conditions far away from the bubble;
R, in resonance.

INTRODUCTION

THE STUDY of oscillations of vapour–gas bubbles in a liquid is of considerable practical interest specifically in regard to the question of the possible use of bubble screens for damping shock waves and the use of acoustic disturbances for intensification of technological processes.

In vapour–liquid flows, the mass, force and energy interactions between phases originate on the interface surfaces. These interactions can significantly alter the flow velocity, pressure and temperature fields. A correct specification of the interphase heat and mass transfer requires the knowledge of interaction of single inclusions with the carrier phase.

At present a number of publications are available in which different aspects of the problem of oscillations of gas bubbles in a liquid are studied. It has been revealed [1] that in the case of small oscillations of a bubble within a wide range of the equilibrium values of its radius the heat transfer dominates over other dissipation mechanisms, i.e. viscosity and compressibility of the liquid. The problem of heat transfer in the course of nonlinear oscillations of a gas bubble was studied experimentally [2]. The results of numerical solution for the nonlinear problem of thermal and dynamic interactions of a gas bubble with the liquid induced by a sudden pressure change in the liquid are presented in [3]. A number of studies deal with the study of growth and collapse of vapour bubbles in a liquid (see, for example, the literature cited in a recent survey [4]). The assumptions of the temperature uniformity in the bubbles and of the thinness of a thermal boundary layer in the liquid, adopted in the majority of these studies, considerably simplify the problem but hold under certain restrictions only.

The heat and mass transfer effect on the vapour bubble dynamics with account for the temperature nonuniformity in it is considered in [5]. Wang [6] and Khabeev [7], while calculating the dynamics of vapour bubbles in a sound field, have revealed the existence of two resonant dimensions of vapour bubbles. The existence of a new resonant frequency of a vapour bubble, which differs from that reported by Minnaert [8], has been also revealed in [7].

Resonant properties of homogeneous equilibrium vapour bubbles were also the concern of Finch and Neppiras [9] who determined the resonant frequencies of a bubble, but not quite correctly. In [10], an attempt was made to analytically determine the resonant dimensions of vapour bubbles and to furnish a physical explanation of the nature of the second resonance. However, neglect of the capillary effects as well as some other inaccuracies have led Hsieh [10] to an erroneous formula which fails to give the realistic values of the resonant dimensions of vapour bubbles, for example, in versions calculated in [6, 7].

This paper gives the results of investigation of the heat and mass transfer effect on the dynamics of vapour–gas bubbles as well as of the reverse effect of the dynamics of radial bubble motion on the enhancement of heat transfer between the bubbles and liquid. These studies were carried out with reference to the analysis of wave processes in vapour–liquid mixtures having a bubble structure.

GOVERNING EQUATIONS

The assumptions adopted are those used in the Rayleigh formulation [11] for the dynamics of a single bubble, viz. a spherical symmetry of the process and pressure uniformity $p_v(t)$ (homobaricity) within the bubble. In the course of bubble oscillations, the homobaricity prevails when the size of the bubble is much less than the length of a sound wave in the gas, $\omega R \ll C$. On the other hand, when the bubble radius changes monotonically (collapse, growth), the homobaricity condition can be written down [12] as

$$(v_\sigma/C)^2 \ll 1.$$

At the same time it is supposed according to the equation of state that the gas density at each point corresponds to its temperature. This statement of the problem, when the pressure uniformity and the temperature and gas density nonuniformities in the bubble are assumed, is valid for a wide range of bubble sizes, since it has been estimated [3] that the characteristic time of temperature equalization in the bubble considerably exceeds the time of pressure equalization.

By introducing the concentration of vapour, k_v, and of the inert gas, k_g, in the vapour–gas mixture the thermophysical parameters of the mixture at each point will be determined as

$$\varphi = k_v\varphi_v + k_g\varphi_g$$

$$k_v = \rho_v/\rho, \quad k_g = \rho_g/\rho, \quad \rho = \rho_v + \rho_g,$$

$$k_v + k_g = 1.$$

In what follows, we shall denote, for simplicity, k_v by k and k_g by $1 - k$.

Within the framework of the assumptions made and the notation adopted, the continuity, state and heat influx equations will have the form

$$\frac{\partial \rho_v}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\rho_v(v + w_v)] = 0$$

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2\rho_g(v + w_g)] = 0$$

$$\rho_g w_g = -\rho_v w_v = \rho D\partial k/\partial r, \quad [0 \le r < R(t)] \tag{1}$$

$$u_g = c_g T, \quad u_v = c_v T, \quad T_v = T_g = T,$$

$$p = p_v + p_g = \rho B T$$

$$\rho_g\frac{du_g}{dt} + \rho_v\frac{du_v}{dt} = \frac{p}{\rho}\frac{d\rho}{dt} + \frac{1}{r^2}\frac{\partial}{\partial r}\times\left(\lambda r^2\frac{\partial T}{\partial r}\right) + \rho D\frac{\partial k}{\partial r}\frac{\partial(u_v - u_g)}{\partial r}$$

$$r^2 v_l = R^2 v_{l\sigma}, \quad u_l = c_l T_l, \quad \rho_l = \text{const}, \quad (R < r < \infty)$$

$$\rho_l\left(\frac{\partial u_l}{\partial t} + v_l\frac{\partial u_l}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_l r^2\frac{\partial T_l}{\partial r}\right) + 12\mu_l\frac{v_l^2}{r^2}.$$

The boundary conditions for the system (1) on the moving boundary $R(t)$, at the centre and at infinity are

$$r = R(t), \quad T_l = T_v = T_\sigma, \quad \lambda_l\partial T_l/\partial r - \lambda\partial T/\partial r = jl$$

$$\rho_v(\dot{R} - v - w_v) = \rho_l(\dot{R} - v_l) = j, \quad \rho_g(\dot{R} - v - w_g) = 0 \tag{2}$$

$$r = 0, \quad \partial T/\partial r = \partial k/\partial r = 0, \quad T_l(\infty) = T_\infty.$$

The kinetics of phase transitions is described by the Hertz–Knudsen–Langmuir equation

$$j = \kappa\frac{p_s(T_\sigma) - p_{v\sigma}}{(2\pi B_v T_\sigma)^{1/2}}. \tag{3}$$

The equation for bubble oscillations in a viscous incompressible fluid in the presence of phase transitions [12] and that for the bubble mass change are

$$R\dot{v}_{l\sigma} + \frac{3}{2}v_{l\sigma}^2 + 2v_{l\sigma}j/\rho_l = \frac{p - p_\infty - 2\sigma/R}{\rho_l} - \frac{4\mu_l}{\rho_l R}v_{l\sigma} \tag{4}$$

$$\dot{m} = 4\pi R^2 j. \tag{5}$$

When the homobaricity condition is satisfied, there is an integral of the heat influx equation for the gas phase

$$\frac{dp}{dt} = \frac{-pR^2 v_\sigma + \int_0^R Gr^2\,dr}{R^3/3 - \int_0^R \frac{B}{c+B} r^2\,dr}$$

$$G = \frac{(B_v - B_g)c_v}{(c+B)} \frac{T}{r^2} \frac{\partial}{\partial r}\left(\rho D r^2 \frac{\partial k}{\partial r}\right)$$

$$+ \frac{B}{c+B}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2 \frac{\partial T}{\partial r}\right) + (c_v - c_g)\rho D \frac{\partial k}{\partial r}\frac{\partial T}{\partial r}\right]. \quad (6)$$

The continuity equation for the vapour–gas mixture, with the use of the homobaricity condition and the boundary condition $v(0, t) = 0$, yields the velocity profile in the bubble

$$v(r,t) = \frac{1}{pr^2}\int_0^r \left(G + \frac{B}{c+B}\frac{dp}{dt}\right) r^2\,dr - \frac{r}{3p}\frac{dp}{dt}. \quad (7)$$

For a one-component bubble, equations (6) and (7) can be considerably simplified

$$\frac{dp}{dt} = \frac{3(\gamma - 1)}{R}\left(\lambda \frac{\partial T}{\partial r}\right)_\sigma - \frac{3\gamma p v_\sigma}{R} \quad (8)$$

$$v(r,t) = \frac{r}{R} v_\sigma + \frac{\gamma - 1}{\gamma p}\left[\lambda \frac{\partial T}{\partial r} - \frac{r}{R}\left(\lambda \frac{\partial T}{\partial r}\right)_\sigma\right]. \quad (9)$$

As is seen, the temperature gradients deviate the velocity profile from a linear one.

For the temperature dependence of the saturated vapour pressure the following approximation [12] has been applied

$$p_s = p_* \exp(-T_*/T_s), \quad (10)$$

where p_* and T_* are found from the condition of the best approximation of tabulated data in the specified pressure range. Ordinary exponential temperature dependences have been employed for the heat conduction and diffusion coefficients.

DISCUSSION OF RESULTS

For nonlinear modes of radial motions of a bubble (oscillations, growth, collapse) the results of numerical solution of the problem are presented in [3, 5, 13] for gas, vapour and vapour–gas bubbles, respectively. The problem was solved using a finite-difference technique by dividing the whole system into spherical layers inside and outside the bubble and employing the variable $\xi = r/R(t)$ which 'freezes' the moving boundary of the bubble. With allowance made for the finite thermal conductivity of liquid, the boundary condition at infinity can be applied to the last layer of the liquid. The calculation is checked by means of a fit of the bubble mass predicted by integrating equation (5) to that obtained by direct layer-to-layer calculation. In the examples considered in [3, 5, 13], the agreement was within $\pm 1\%$.

Calculations [3] for a gas bubble indicate that even on its strong compression, when the gas temperature at its centre amounts to about 1000 K, the temperature of the bubble surface stays practically constant and is equal to the temperature of the surrounding liquid which acts as a thermostat, i.e. for a gas bubble the internal thermal problem plays a major role, while the boundary condition on the bubble surface may then be set as $T_\sigma = T_0$. There are, however, some publications, see for example [14], in which heat transfer between gas bubbles and liquid is wrongly believed to be governed by thermal resistance of the liquid.

Figure 1 shows the characteristic temperature distribution inside a gas bubble oscillating in consequence of a sudden pressure increase from p_0 to p_l in the liquid far from the bubble — a situation corresponding to the behaviour of bubbles in the front part of the bubble screen upon entrance of a shock wave. The initial data are as follows: $R_0 = 1$ mm, $p_0 = 1$ bar, $p_l = 2$ bar. Curves 1–7 correspond to the instants of time: $\omega t = 0$; $2\pi/5$; $3\pi/5$; π; $6\pi/5$; $8\pi/5$; 2π, with $\omega t = 0$ and 2π corresponding to two successive moments of the greatest compression of bubble. It is interesting that over some time intervals, for example $2\pi/5 \leq \omega t \leq 6\pi/5$, the heat flux is directed to the bubble inwards, although the mean temperature of the gas in the bubble $\langle T \rangle$ is above the liquid temperature T_0. In this case, the formally determined dimensionless heat flux, viz. the Nusselt number, is negative

$$Nu = \frac{2Rq_\sigma}{\lambda_g(\langle T \rangle - T_0)}, \quad q_\sigma = -\lambda_g \frac{\partial T_g}{\partial r}\bigg|_{r=R} \quad (11)$$

where q_σ is the heat flux from the bubble into the liquid. The reason for this behaviour is that, being heated on compression, the bubble gives up its heat to the liquid intensely due to a large gas–temperature gradient in a thin wall layer and a high absorption of this heat by the liquid, while during bubble expansion the heat conduction does not succeed in compensating for cooling of the gas wall layers caused by expansion.

The results of calculations [5] have shown that at moderate velocities of the bubble surface the phase

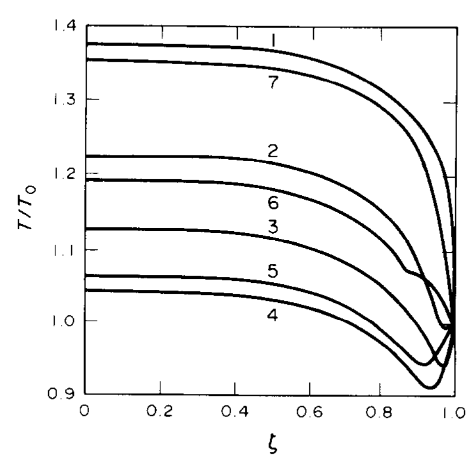

FIG. 1. Characteristic temperature distribution within an oscillating gas bubble at different instants of time: 1, $\omega t = 0$; 2, $2\pi/5$; 3, $3\pi/5$; 4, π; 5, $6\pi/5$; 6, $8\pi/5$; 7, 2π.

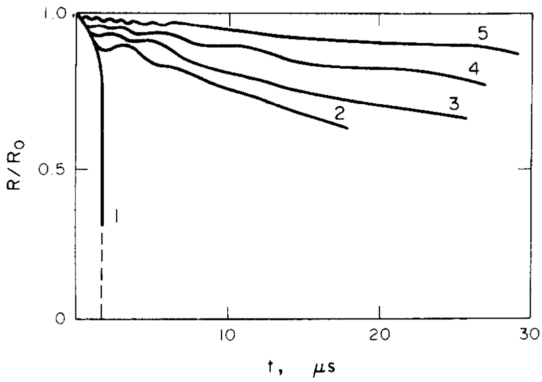

FIG. 2. Radius vs time curve for collapse of a vapour bubble in various liquids under identical conditions: $R_0 = 10\,\mu\text{m}$, $p_0 = 1$ bar, $p_l = 1.2$ bar. Curves 1–5 are for water, Freon-12, liquid nitrogen, hydrogen and helium, respectively.

transition virtually follows the quasi-equilibrium scheme $T_\sigma = T_s(p_v)$ (there is no delay due to the kinetics of phase transitions). With a step-wise variation of pressure in the liquid both monotonic and oscillatory changes in the vapour bubble parameters are possible.

Figure 2 presents the calculated radius vs time curves for condensational collapse of vapour bubbles in various liquids. Curves 1–5 show the behaviour of bubbles in water, Freon-12, nitrogen, hydrogen, and helium under identical initial conditions. The pressure in the liquid was instantly brought up from $p_0 = 1$ bar to $p = 1.2$ bar, the initial bubble radius was $10\,\mu$m ($R_0 = 10\,\mu$m), the initial temperature in the system was uniform and equal to its own saturation temperature corresponding to the equilibrium pressure in the bubble, $T_0 = T_s(p^0)$, $p^0 = p_0 + 2\sigma/R_0$.

The course of curves 1–5 in Fig. 2 confirms the effectiveness of the parameter

$$\Phi = Ja^2 a_l/R_0(\rho_l/\Delta p)^{1/2}$$

suggested in [15] to predict the nature of collapse of vapour bubbles. Here $Ja\ (=c_l\,\Delta T\rho_l/l\rho_{v0})$ is the Jacob number, $\Delta p = p_l - p_0$, $\Delta T = T_s(p_l) - T_0$. At large Φs, the process of bubble collapse is similar to the limiting inertial regime, while at small Φs, to the thermal regime. For curves 1–5 in Fig. 2 there are the following values of Φ: 8; 6×10^{-2}; 10^{-2}; 5×10^{-4}; 2×10^{-4}.

Figure 3 shows a comparison between the predicted radius–time curves and the experimental data of [15] on collapse of vapour–air bubbles in water for

$$R_0 = 3.66\,\text{mm},\quad p_0 = 0.636\,\text{bar},\quad T_0 = T_s(p_0) \qquad \text{(curve 1)}$$

$$R_0 = 3.36\,\text{mm},\quad p_0 = 0.734\,\text{bar},\quad T_0 = T_s(p_0). \qquad \text{(curve 2)}$$

In both cases, the system was instantly exposed to the atmospheric pressure $p_l = 1$ bar. The initial content of a nonsoluble gas in both cases amounted to $k_g = 0.0002$ and $k_g = 0.0006$, respectively. This small amount of gas does not virtually influence the initial behaviour of the radius–time curve leading to incomplete collapse of bubbles only. To make the picture more lucid, the curves are plotted on different scales: the left vertical and the upper horizontal axes correspond to curve 1, the right vertical and the lower horizontal, to curve 2. A good agreement between the prediction and experiment is evident. The dashed curve represents theoretical calculations [16] for this experiment. In these calculations, it was arbitrarily assumed that velocity distribution of vapour particles in the bubble was parabolic and this resulted in distortion of the temperature profile. In actual fact, the velocity profile in a vapour bubble is described by equation (9). The difficulty posed by determination of the velocity profile in a bubble was responsible for nonjustifiable omission of the convective term in the heat conduction equation by some authors or its artificial pre-assignment (linear [17] or parabolic [16]), by others. Cho and Seban [16] also ignored the heat flux into the vapour phase. But the main drawback of [16], which had led to a substantial discrep-

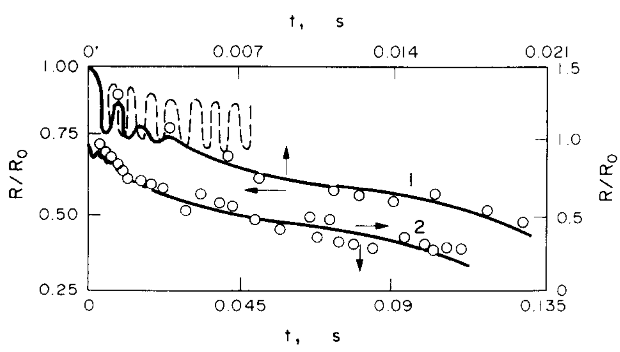

FIG. 3. Comparison of the calculated 'radius–time' curves with the experimental data of [15] and the theoretical calculations of [16] (dashed curve) for vapour–air bubbles collapsing in water.

ancy with the experiment, was the wrong step in the finite-difference scheme for the liquid energy equation.

The step of the finite difference grid for the liquid energy equation should apparently satisfy the condition $h_l \ll \delta_l$, where $\delta_l \sim (a_l/\omega)^{1/2}$ is the thermal boundary layer thickness in the liquid, $\omega = (3\gamma p_0/\rho_l)^{1/2} R_0^{-1}$ is the frequency of oscillations. For the version presented in Fig. 3 $\delta_l \sim 10^{-5}$ m or $\delta_l \sim 0.3 \times 10^{-2} R_0$. Consequently, the step h_l should satisfy the condition $h_l \lesssim 10^{-3} R_0$. A larger step, $h_l = 10^{-2} R_0$, selected by Cho and Seban [16], has led to substantial underestimation of the liquid temperature gradient in the wall boundary layer and, as a consequence, to a highly underestimated phase transition rate. Thus it happened that $j/\rho_v \ll \dot{R}$ and, according to (2), that $v_\sigma \approx \dot{R}$. The latter, once the heat flux into the vapour phase is neglected [16], leads, via integration of equation (8), to the familiar relation for an adiabatic gas bubble

$$p_v R^{3\gamma} = \text{const.} \tag{12}$$

It is hardly surprising on that account that Cho and Seban [16] have obtained a close coincidence of their predictions with the behaviour of an adiabatic constant mass gas bubble and a qualitative discrepancy with the experiments of [15] in which clearly defined oscillations of bubbles were not observed.

It should be noted that consideration of the effect of heat and mass transfer on the vapour–gas bubble dynamics under the assumption that the bubble temperature is uniform [18] is permissible only at sufficiently low concentrations of gas in the bubble when heat transfer between the bubble and the liquid is governed by the thermal resistance of the latter. At large gas contents in the bubble, heat transfer is controlled by the thermal resistance of the bubble.

The internal thermal problem becomes also significant at high parameters when the thermal properties of vapour and liquid get closer.

Note that the temperature of vapour in a bubble is virtually uniform and equal to the saturation temperature not only under the condition, which is usually employed but rarely realized in practice, that the bubble size is less than the thickness of a thermodiffusional layer in vapour, $R < (a_v/\omega)^{1/2}$, but also at $c_p T_s/l \approx 1$ or $c_s \approx 0$, where c_s is the vapour heat capacity along the phase equilibrium curve [19]

$$c_s = c_p - T\frac{\mathrm{d}p}{\mathrm{d}T}\frac{\partial}{\partial T}\left(\frac{1}{\rho}\right)_p = c_p - \frac{l}{T_s}.$$

For liquid helium $c_s \approx 0$ at atmospheric pressure, while for water $c_s \approx 0$ at $p \sim 30$ bar.

SMALL OSCILLATIONS

Thermal effects in the course of small free and forced oscillations of gas bubbles were considered in [1, 20, 21]. The effects of heat transfer and of nonequilibrium phase transitions during oscillations of bubbles were considered in [7].

Under the assumption that the acoustic pressure amplitude p_A is small as compared with the static pressure p_∞ in the liquid

$$p(\infty) = p_\infty + p_A \exp(i\omega t)$$

and that the bubble radius is described by the real part of the expression

$$R = R_0[1 + \alpha \exp(i\omega t)], \quad |\alpha| \ll 1$$

Khabeev [7] has obtained an analytical solution of the problem which, in a quasi-equilibrium approximation, is of the form

$$\alpha_p = p_\infty/S, \quad \alpha_p = \alpha p_\infty/p_A, \tag{13}$$
$$S = \rho_l \omega^2 R_0^2 + 2\sigma/R_0 - 4i\omega\mu_l - 3/\Psi.$$

Here S is the resonant function, Ψ is the bubble compressibility

$$\Psi = \frac{1 - ia_v[B_1 G_1 + F(1 + Ez^{1/2})]/\omega R_0^2}{\gamma p_0 + i\omega a_v \rho_{v0}[B_1 G_1 b/(1-b) - F(1 + Ez^{1/2})]/3}$$
$$b = c_p T_0/l, \quad z = i\omega R_0^2/a_v, \quad E = (a_v/a_l)^{1/2},$$
$$B_1 = z^{1/2}\coth z^{1/2} - 1, \quad p_0 = p_\infty + 2\sigma/R_0,$$
$$T_0 = T_s(p_0), \quad G_1 = 3(\gamma - 1)(1-b)^2,$$
$$F = 3(\gamma - 1)b^2 \lambda_l/\lambda_v. \tag{14}$$

Analysis of equation (13) reveals that

$$\lim_{R_0\to 0}|\alpha_p| = \lim_{R_0\to\infty}|\alpha_p| = \lim_{\omega\to\infty}|\alpha_p| = 0,$$
$$\lim_{\omega\to 0}|\alpha_p| = p_\infty R_0/2\sigma \tag{15}$$
$$\lim_{R_0\to 0}\beta = \lim_{R_0\to\infty}\beta = \lim_{\omega\to 0}\beta = \lim_{\omega\to\infty}\beta = 0$$

where β is the phase shift between oscillations of the bubble radius and pressure at infinity. At $\sigma = 0$

$$\lim_{R_0\to 0}|\alpha_p| = p_\infty/4\omega\mu_l, \quad \lim_{\omega\to 0}|\alpha_p| = \infty. \tag{16}$$

It follows from equation (15) that at $\sigma \neq 0$ for any finite frequency there is at least one such bubble size at which $|\alpha_p|$ attains the maximum value. This proves the invalidity of the results of [9] which indicate that there are no resonant vapour bubbles at sufficiently high frequencies of the acoustic field.

In the absence of phase transitions, the analysis of equation (13) shows that

$$\lim_{R_0\to 0}\beta = \lim_{\omega\to 0}\beta = \pi, \tag{17}$$
$$\lim_{\omega\to 0}|\alpha_p| = \frac{1}{3\gamma + 2\sigma(3\gamma - 1)/R_0 p_\infty}.$$

Note that for a gas bubble of constant mass Macedo and Yang [20] have obtained

$$\lim_{R_0\to 0}\beta = \lim_{\omega\to 0}\beta = 0.$$

This, as well as the existence of two resonant dimensions of gas bubbles, which has been established in [20], is true only at artificially fixed pressure in the bubbles, $p_0 = p_\infty + 2\sigma/R_0 = \text{const}$. But then the static pressure in the liquid, p, would decrease with decreasing R_0 and for small bubbles (in water at $R_0 \lesssim 1\ \mu\text{m}$) should become negative. Such a 'resonance' would then be realized at a negative pressure in the liquid, $p_\infty \approx -2p_0$, which is impracticable.

The expression for the bubble compressibility can be considerably simplified if it is remembered that for the majority of substances the following estimates are valid in a wide range of variable parameters

$$0 < G_1 < 1, \quad 0 < b \lesssim 1, \quad F \sim 1, \quad a_l \ll a_v \tag{18}$$

$$\omega \rho_{v0} a_v \frac{F}{p_0} |1 + Ez^{1/2}| \ll 1, \quad |B_1| \ll F|1 + Ez^{1/2}|.$$

After simplification we get

$$\Psi = \frac{1}{\gamma p_0} \frac{z + F(1 + Ez^{1/2})}{z}. \tag{19}$$

The resonant frequency is determined by solving the following equation

$$\partial |S| / \partial \omega = 0 \tag{20}$$

and checking the condition $\partial^2 |S| / \partial \omega^2 > 0$. For rather large bubbles, when the condition

$$|z| \gg F|1 + Ez^{1/2}| \tag{21}$$

is satisfied, the effect of heat and mass transfer on their dynamics is small. The solution of equation (20) may then be sought in the form

$$\omega_R = \omega_0(1 + \varepsilon), \quad |\varepsilon| \ll 1 \tag{22}$$

where $\omega_0 = (3\gamma p_0/\rho_l)^{1/2} R_0^{-1}$ is the natural frequency of an adiabatic gas bubble in an ideal liquid [8]. By substituting (13) into (20), with account for (19) and (21), and using the smallness, ε, of capillary and viscous effects, we obtain

$$\varepsilon = \frac{-Fa_v}{2\sqrt{[(2a_l)(3\gamma p_0/\rho_l)^{1/4} R_0^{1/2}]}}. \tag{23}$$

The correction obtained for the resonant frequency of large vapour bubbles characterizes the effect of heat and mass transfer processes on the dynamics of bubbles. Of course, it increases with a decreasing R_0. In the other limiting case of rather small bubbles, when the following conditions are satisfied

$$|z| \ll F|1 + Ez^{1/2}|, \quad E|z^{1/2}| \gg 1, \quad \rho_l \omega^2 R_0^2 \ll \frac{2\sigma}{R_0},$$
$$4\omega\mu_l \ll AR_0\omega^{1/2} \tag{24}$$

the expression for the resonant function is of the form

$$S = \frac{2\sigma}{R_0} - AR_0\omega^{1/2}(1 + i), \quad A = \frac{l^2 \rho_{v0}^2}{\lambda_l T_0} \sqrt{\left(\frac{a_l}{2}\right)}. \tag{25}$$

In [9, 10], the resonant frequency of a vapour bubble is determined by solving the equation

$$\text{Re}(S) = 0. \tag{26}$$

This approach is incorrect since, besides the real part, the imaginary part of the resonant function is also a function of the bubble radius and of the acoustic field frequency. Determination of the resonant frequency from equation (26), giving an illusion that there is a universal dependence relating the resonant frequency to the bubble size, does not lead to great errors in [9] only in the region of large bubble radii and small acoustic field frequencies when the formula of Minnaert is valid. This very inaccuracy has led the author of [10] to an erroneous formula which relates the resonant frequency of a bubble to its radius. Moreover, neglect of the capillary effects [10, 22] precludes the very possibility of the second resonance.

When the conditions (24) are satisfied, it is possible to obtain a simple formula relating the resonant frequency of a vapour bubble to its radius. By solving for (25) equation (20) in ω we obtain

$$\omega_R = (\sigma/A)^2 R_0^{-4}. \tag{27}$$

For the first time formula (27) was derived in [23]. Later a formula of the same type was published in [24].

Provided the condition $2\sigma/R_0 \lesssim p_\infty$ is also satisfied, then by solving, for (25), the equation

$$\partial |S| / \partial R_0 = 0 \tag{28}$$

we obtain the dependence of the resonant size of a bubble on the acoustic field frequency

$$R_R^4 = 2(\sigma/A)^2 \omega^{-1}. \tag{29}$$

This relation is not exactly the reverse of (27) as it differs by the numerical factor.

Note that if we set $\sigma = 0$, then neither the resonant frequency, nor the resonant size of the bubble will exist in the range considered. In fact, in this case

$$S = \rho_l \omega^2 R_0^2 - AR_0\omega^{1/2}(1 + i).$$

It can be easily verified that then equations (20) and (28) will have no real roots. At the same time, determination of the resonant frequency within the framework of the approach of [9, 10] via the solution or equation (26) leads in this case to an erroneous conclusion on the existence of the resonant frequency.

By evaluating the range of dimensions of bubbles and acoustic field frequencies for which equations (27) and (29) can hold we shall obtain that for water at $p_\infty = 1$ bar these estimates yield

$$10^{-5}\ \text{m} \lesssim R_0 \lesssim 10^{-4}\ \text{m}, \quad 10\ \text{Hz} \lesssim f \lesssim 10^4\ \text{Hz}.$$

If the conditions

$$|z| \ll F, \quad E|z^{1/2}| \ll 1$$
$$\rho_l \omega^2 R_0^2 \ll \frac{2\sigma}{R_0}, \quad 4\mu_l \ll AR_0^2 \sqrt{\left(\frac{2}{a_l}\right)} \tag{30}$$

are satisfied, the expression for the resonant frequency

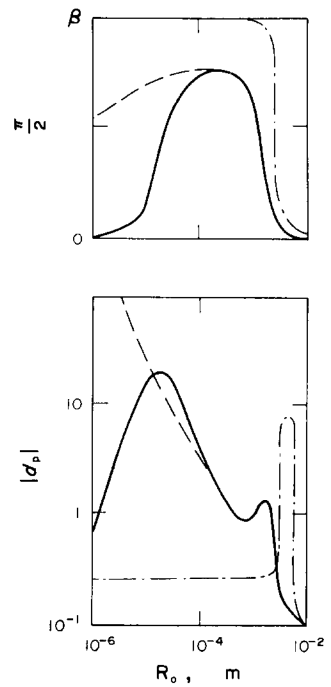

FIG. 4. Dependence of the amplitude and phase of oscillations on the radius of a vapour bubble during its oscillations in water at a frequency of $f = 1$ kHz. Dashed curve corresponds to $\sigma = 0$, dashed-dotted curve, to a gas bubble.

has the form

$$S = (2\sigma/R_0) - AR_0^2\omega\sqrt{(2/a_l)}[\sqrt{(\omega/2a_l)} + i]. \quad (31)$$

Solution of equation (20) for (31) gives the following dependence of the resonant frequency of a bubble on its radius

$$\omega_R = \frac{9\sigma^2}{4A^2R_0^4} \quad (32)$$

which differs from (27) by a numerical factor only. The estimates show that for water at $p_\infty = 1$ bar equation (32) is valid for bubbles having $R_0 \gtrsim 10^{-4}$ m. However, the Q-factor of the second resonance in this region is much below the Minnaertian one.

Figure 4 presents the dependence of the dimensionless amplitude of oscillations $|\alpha_p|$ and of phase β on the bubble radius in the course of its oscillations in water at atmospheric pressure and frequency of 1 kHz. Dotted and dash-dotted curves correspond to the cases $\sigma = 0$ and $j = 0$. Here, the second resonance is absent.

The calculations have shown that for large vapour bubbles (in water at $p_\infty = 1$ bar for $R_0 \gtrsim 1$ mm) the Minnaertian resonance alone is virtually observed (the second resonance does not show itself in this region because of the low Q-factor), while for the bubbles having $R_0 \lesssim 0.1$ mm there is only a resonance which is associated with capillary effects and phase transitions. In the intermediate region, the response function $|\alpha_p|$ has two local maxima, with the second appearing at

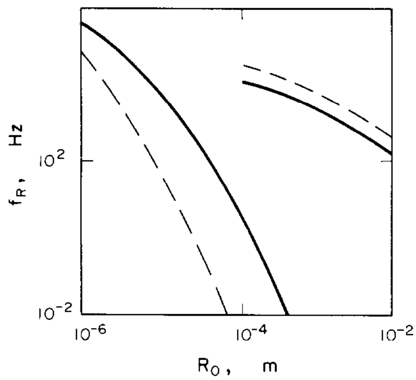

FIG. 5. Dependence of the resonant frequency of a vapour bubble on its radius in water and nitrogen boiling at atmospheric pressure (solid and dashed lines, respectively).

very low frequencies of the acoustic field. The position of resonances agrees well with the foregoing formulae.

Figure 5 shows the plots of the resonant frequency of a vapour bubble in water and liquid nitrogen (solid and dashed lines, respectively) vs its radius which were calculated by equation (13) at $p_\infty = 1$ bar. Both curves have 2 branches. The second branch in the region of small R_0s appears only then when the capillary effects and phase transitions are accounted for simultaneously. In the region $R_0 \gtrsim 10^{-5}$ m the predicted relations agree well with the approximations (22), (27) and (32). The dependence of the resonant bubble size on the acoustic field frequency in a wide range of frequencies is a double-valued function. For $\sigma = 0$, the resonant characteristics of vapour bubbles in boiling water and a number of cryogenic liquids are calculated in [22].

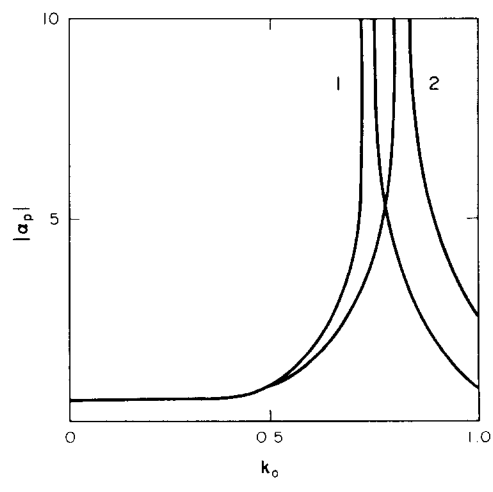

FIG. 6. Dependence of the amplitude of oscillations of a vapour–air bubble in water at a frequency of $f = 18$ kHz on equilibrium vapour concentration. Curves 1 and 2 correspond to $R_0 = 1\ \mu$m and $3\ \mu$m, respectively.

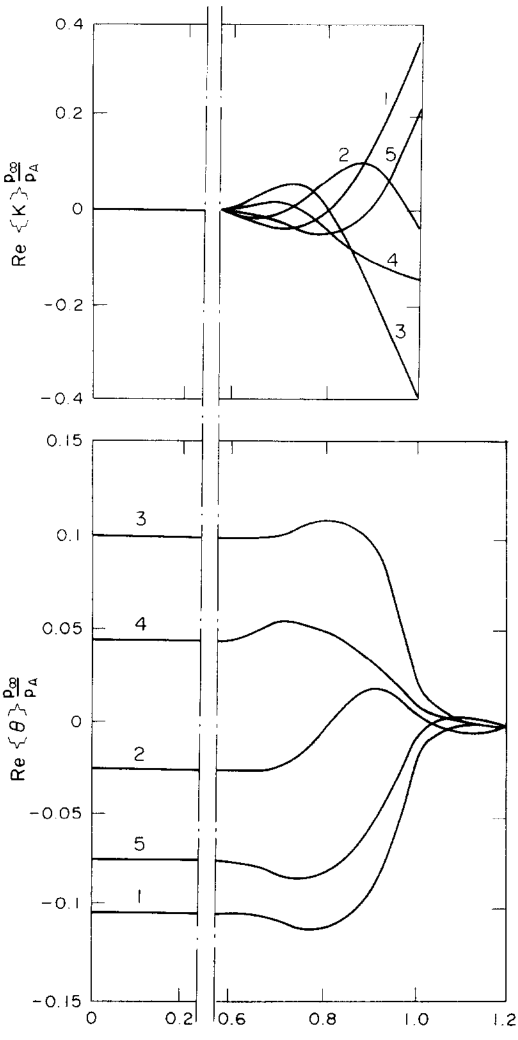

FIG. 7. Concentration and temperature distributions during oscillations of a vapour–air bubble in water exposed to a sound field. $R_0 = 210\,\mu$m, $k = 0.8$, $p_\infty = 1$ bar, $f = 20$ kHz. Curves 1–5 correspond to $\omega t = 0$, $2\pi/5$, $4\pi/5$, $6\pi/5$, $8\pi/5$.

The existence of two resonances of vapour bubbles is due to the dispersion of frequencies since compressibility of small vapour bubbles, in contrast to the gas ones, depends greatly on the frequency of oscillations. This also explains the fact that a vapour bubble, unlike a gas one, oscillates at low frequencies in phase with the liquid pressure far away from the bubble.

In the case of a vapour–gas bubble its compressibility depends, among other things, on the vapour concentration in the bubble. As is shown in [25], there are then the critical values of vapour concentration in bubbles at which the response function of a vapour–gas bubble can increase markedly. Figure 6 shows the amplitude of oscillations of vapour–gas bubbles in water at atmospheric pressure as a function of the equilibrium vapour concentration. Curves 1 and 2 correspond to $R_0 = 1\,\mu$m and $3\,\mu$m. The acoustic field frequency is $f = 18$ kHz. It is seen that the response function can be nonmonotonic with respect to k_0.

Figure 7 presents temperature and concentration distributions in the course of oscillations of a vapour–air bubble of radius $R_0 = 210\,\mu$m in water at atmospheric pressure and frequency of 20 kHz. The initial equilibrium vapour concentration was $k_0 = 0.8$. Curves 1–5 are for $\omega t = 0$; $2\pi/5$; $4\pi/5$; $6\pi/5$; $8\pi/5$; with $\omega t = 0$ corresponding to the instant of the greatest bubble expansion. The quantities θ and K are nondimensional amplitudes of the temperature and concentration fluctuations

$$T = T_0[1 + \theta(r)\exp(i\omega t)],$$
$$k = k_0[1 + K(r)\exp(i\omega t)].$$

Unlike the temperature, the concentrations of components in the central part of vapour–gas bubbles do not fluctuate during oscillations of bubbles. Since for an air–water vapour mixture at $p_\infty \sim 1$ bar the Lewis parameter $Le = a/D \sim 1$, the thicknesses of the thermal and diffusional boundary layers are about the same.

In the case of small free oscillations of a vapour bubble, its radius can be described by the real part of the expression

$$R = R_0[1 + \delta\exp(ht)], \quad |\delta| \ll 1.$$

The condition of the existence of a nontrivial solution for the system leads in this case to a transcendental equation in h which, for the quasi-equilibrium bubbles in the states far from the critical ones when $\rho_v \ll \rho_l$, is of the form [7]

$$H + \frac{3\gamma NH}{H^2 - N\Sigma} + M = 0,$$

$$M = F(1 + EH^{1/2}) + G_1(H^{1/2}\coth H^{1/2} - 1), \quad (33)$$

$$H = \frac{hR_0^2}{a_v}, \quad N = \frac{p_0 R_0^2}{\rho_l a_v^2}, \quad \Sigma = \frac{2\sigma}{R_0 p_0}.$$

With the use of the argument principle [26] it is possible to show that account for the temperature nonuniformity in the bubble, even in the absence of phase transitions when equation (30) can be simplified to

$$H + 3\gamma NH/(H^2 - N\Sigma) + 3(\gamma - 1)(H^{1/2}\coth H^{1/2} - 1) = 0 \quad (34)$$

results in the situation that this equation has an infinite number of roots in the left half plane ($\mathrm{Re}\, h < 0$). This is attributable to periodicity of the cotangent function which describes temperature distribution in the bubble [1, 7] and enters equation (33). It can be shown however that all of the roots of equation (33) except for the two, complex conjugate, ones are real and exceed the real part of the complex roots in absolute magnitude, thereby showing the correctness of results of [1, 7] where the characteristic equations were solved numerically without analysing the number and structure of the roots.

In the case of a homogeneous vapour bubble, equation (33) can be reduced to the polynomial

$$x^6 + FEx^5 + Fx^4 + (3\gamma - \Sigma)Nx^2 - FEN\Sigma x - FN\Sigma = 0, \quad x = H^{1/2}. \quad (35)$$

The analysis shows that equation (35) in the left half plane has only a pair of complex conjugate roots, i.e. that a homogeneous vapour bubble has only one natural frequency. At the same time, in the case of forced oscillations within a certain range of bubble dimensions the response function of a homogeneous vapour bubble has local maxima at two frequencies of the acoustic field. It should be noted, however, that the Q-factor of the second resonance is much less than the Minnaertian one.

For rather large bubbles, an asymptotic solution to equation (33) is obtainable.

The expressions for the natural frequency of a vapour bubble and logarithmic decrement of its oscillations induced by heat and mass transfer are

$$\omega = \frac{v_0}{R_0}\left[1 - \frac{G_1}{2\sqrt{(Pe_v)}} - \frac{FE^2}{2\sqrt{(Pe_l)}}\right],$$

$$\Lambda = \pi\left[G_1\frac{\sqrt{(Pe_v - 2)}}{Pe_v} + FE^2\frac{\sqrt{(Pe_l + 2)}}{Pe_l}\right] \quad (36)$$

$$v_0 = \sqrt{\left(\frac{3\gamma p_0}{\rho_l}\right)}, \quad Pe_i = \frac{2R_0 v_0}{a_i} \quad (i = v, l).$$

Formulae (36) hold whenever $\Lambda < 1$, $|\varepsilon| \ll 1$ or

$$Pe_v \gg 1, \quad Pe_l^{1/2} \gg FE^2.$$

In the expression (36) for the damping ratio of oscillations of a vapour bubble, the first term is due to thermal resistance of vapour in the bubble, while the second, to thermal resistance of liquid. In the states far from the critical ones, the second term greatly exceeds the first one. With no phase changes ($l = \infty$), the second term is absent and formula (36) yields the following expression for the thermal damping ratio of oscillations of a gas bubble

$$\Lambda = \frac{3(\gamma - 1)\pi}{Pe_g}(Pe_g^{1/2} - 2), \quad Pe_g = \frac{2R_0 v_0}{a_g} \gg 1. \quad (37)$$

Here Pe_i ($i = v, g, l$) are the Peclet numbers in which the velocity is taken to be a characteristic radial velocity of small free oscillations of an adiabatic bubble, v_0.

In a similar manner, the solution to equation (34) can be obtained for the other limiting case of very tiny gas bubbles that oscillate in a mode close to the isothermal one with the frequency

$$\omega = Jmh \approx (3p_0/\rho_l)^{1/2} R_0^{-1}.$$

In this case the expression for the damping ratio of oscillations of a bubble is of the form

$$\Lambda = \frac{(\gamma - 1)\pi}{30\gamma}Pe, \quad Pe = \frac{2R_0(3p_0/\rho_l)^{1/2}}{a_g} \ll 1. \quad (38)$$

EFFECTIVE COEFFICIENTS OF HEAT TRANSFER BETWEEN BUBBLES AND LIQUID

The existing literature on the dynamics and heat and mass transfer of vapour–gas bubbles (see, for example, survey [4] and an accompanying list of references) deals in the main with the effect of heat and mass transfer on the dynamics of bubbles. However, interest attaches also to the reverse problems, viz. the effect of dynamics, and of radial oscillations in particular, on the enhancement of heat and mass transfer between the bubbles and the liquid. It is shown in [27] that thermal effects play a major role in the formation of wave structure in a liquid containing gas bubbles. The calculation of two-phase flows, transient ones in particular, with account for nonuniform temperature distribution in the phases is yet a very hard task and requires extensive computer time. An example of this type of calculation is given elsewhere [28]. Therefore, calculation of vapour–liquid mixture flows having a bubble structure faces the main problem which amounts to assigning the coefficients of the interphase interaction, thermal one in particular, within the framework of the two-temperature model that would be valid for a certain class of process.

A formally determined Nusselt number (11) indicates that because of the temperature 'pits' and 'humps' produced in the gas and liquid by oscillations of bubbles the instantaneous values of the Nusselt number can be negative over certain time intervals with a resulting question as to the means of their averaging. It seems natural that the effective heat transfer coefficients be selected from the condition that these should ensure the same heat dissipation as that of the exact solution.

The expression for the damping ratio of free oscillations of a vapour bubble derived by solving the problem within the framework of the three-temperature model (bubble–interface surface–liquid) is of the form

$$\Lambda = \pi\left(G_1\frac{Nu_v}{Pe_v} + FE^2\frac{Nu_l}{Pe_l}\right). \quad (39)$$

By equating the damping ratio components, which are due to thermal resistance of the liquid and the bubble, with the corresponding components from the exact solution (36), we shall obtain the following formulae for the effective coefficients of heat transfer between a radially oscillating vapour bubble and the liquid

$$Nu_v = Pe_v^{1/2} - 2, \quad Pe_v \gg 1$$
$$Nu_l = Pe_l^{1/2} + 2, \quad Pe_l^{1/2} \gg FE^2. \quad (40)$$

Whenever $Pe_l \ll 1$, the effect of the dynamics of

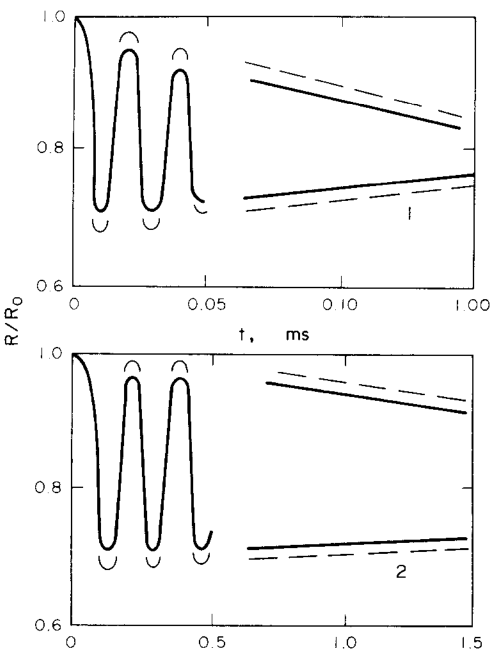

FIG. 8. Dependence of air bubble radii on time. Solid curves, exact solution; dashed curves, solution within the framework of a two-temperature model.

oscillations is small and equation (40) yields $Nu_l = 2$, which is the well-known steady-state solution for a sphere [29]. An expression obtained in a similar way for the effective coefficient of heat transfer between an oscillating gas bubble and the liquid is of the form

$$Nu = Pe_g^{1/2} - 2, \quad Pe_g \gg 1. \tag{41}$$

In the case of rather small gas bubbles oscillating in the mode close to the isothermal one the expression for the damping ratio of its oscillations obtained within the framework of the two-temperature model is

$$\Lambda = \frac{(\gamma - 1)\pi Pe}{3\gamma Nu}, \quad Pe \ll 1. \tag{42}$$

By equating expressions (38) and (42) we obtain that for small gas bubbles

$$Nu = 10.$$

Formula (41) presupposes that a bubble oscillates with the frequency close to the natural frequency of an adiabatic gas bubble. However, in rather strong shock waves the bubbles can oscillate with a frequency differing from the Minnaertian one, while in weak waves the parameters in the wave can change monotonically [27]. A change in the bubble radius in the front part of a smeared wave is described by the formula

$$R = R_0(1 - \delta \exp \varepsilon t), \quad \varepsilon > 0. \tag{43}$$

The expression for the Nusselt number in this case is [30]

$$Nu = 2R(\varepsilon/a_g)^{1/2}, \quad R \gg (a_g/\varepsilon)^{1/2}. \tag{44}$$

Formula (44) can be restated if it is taken into account that

$$\varepsilon = \dot{R}/\Delta R. \tag{45}$$

In the case of rather large gas bubbles a change in the bubble size, ΔR, can be approximately related to its temperature variation on adiabatic compression

$$\Delta R = -\frac{R}{T}\frac{\Delta T}{3(\gamma - 1)}. \tag{46}$$

By substituting (45) and (46) into (44) we shall obtain the following expression for the Nusselt number

$$Nu = 2\left|\frac{R\dot{R}(3\gamma - 1)T}{a_g \Delta T}\right|^{1/2}. \tag{47}$$

Of interest is the fact that by introducing the correction factor

$$K_1 = [2R(3\gamma p/\rho_l)^{1/2}/a_g]^{1/4}/(6\gamma)^{1/2}$$

into formula (47), obtained for exponential compression of gas bubbles, this formula, in the case of small oscillations of bubbles, will give the same damping of oscillations within the framework of the two-temperature model as that given by the exact solution. Note that for air bubbles in water within practically interesting ranges of pressure and bubble sizes $K_1 \sim 1$.

Figure 8 shows the radius–time curves that illustrate the behaviour of air bubbles of two dimensions: (a) $R_0 = 0.1$ mm and (b) $R_0 = 1$ mm in water when the pressure of liquid far from the bubble was suddenly raised from 1 to 2 bar. Solid curves indicate a numerical solution of the complete system of equations that describe heat transfer and gas bubble dynamics in the liquid. Dashed lines represent the solution within the framework of the two-temperature model with the use of the Nusselt number determined by formula (47).

The numerical calculations have shown that formula (47) rather adequately describes heat transfer between the gas bubbles and the liquid in different modes of their radial motion.

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DYNAMIQUE ET TRANSFERT DE CHALEUR ET DE MASSE DE BULLES DE VAPEUR-GAZ DANS UN LIQUIDE

Résumé—Un problème non linéaire d'interaction dynamique, massique et thermique entre une bulle vapeur–gaz et un liquide est considéré en tenant compte de la non-uniformité de la température dans la bulle et de l'interdiffusion des composants du mélange vapeur–gaz. Une solution numérique est obtenue pour le mouvement radial de la bulle induit par un changement brusque de pression dans le liquide, situation qui correspond en particulier au comportement des bulles lorsqu'un front d'onde de choc entre dans un rideau de bulles. On considère aussi les bulles de vapeur–gaz qui oscillent dans le liquide sous l'influence d'un champ sonore. Les effets de capillarité et les transitions de phase, considérés ensemble, produisent une fréquence de résonnance pour les petites bulles de vapeur, différemment de ce qui est décrit par Minnaert. On obtient des expressions pour la fréquence et l'amortissement thermique des oscillations de bulles. On détermine les coefficients effectifs de transfert thermique entre le liquide et des bulles qui oscillent radialement.

DYNAMIK, WÄRME- UND STOFFAUSTAUSCH VON DAMPF-GASBLASEN IN EINER FLÜSSIGKEIT

Zusammenfassung—Es wird ein nichtlineares Problem von thermischer, stofflicher und dynamischer Wechselwirkung von Dampf-Gasblasen und einer Flüssigkeit unter Berücksichtigung der Temperaturungleichförmigkeit in der Blase und der wechselseitigen Diffusion der Komponenten des Dampf-Gasgemisches betrachtet. Für das Problem der radialen Blasenbewegung, hervorgerufen durch eine plötzliche Druckänderung in der Flüssigkeit, wird eine numerische Lösung erhalten, ein Sachverhalt der insbesondere dem Verhalten von Blasen hinter einer Stoßwellenfront entspricht, wenn letztere ein Blasengebiet durchläuft. Weiterhin werden Dampfblasen betrachtet, die in der Flüssigkeit unter dem Einfluß von Schallfeldern schwingen.

Es wird gezeigt, daß die Oberflächeneffekte und die Phasenübergänge zusammengenommen, eine neue Resonanzfrequenz kleiner Dampfblasen zur Folge haben, die sich von der jenigen unterscheidet, die Minnaert angibt. Der Ausdruck für die Frequenz und das thermische Dämpfungsmaß der Blasenschwingung wurde erhalten. Die zwischen den radial schwingenden Blasen und der Flüssigkeit wirksamen Wärmeübergangskoeffizienten wurden bestimmt.

ДИНАМИКА И ТЕПЛОМАССООБМЕН ПАРОГАЗОВЫХ ПУЗЫРЬКОВ В ЖИДКОСТИ

Аннотация — Рассмотрена нелинейная задача о тепловом, массовом и динамическом взаимодействии парогазового пузырька с жидкостью с учетом неоднородности температуры в пузырьке и взаимной диффузии компонент парогазовой смеси. Приведены результаты численного решения задачи о радиальном движении пузырька, вызванном внезапным изменением давления в жидкости, что, в частности, соответствует поведению пузырьков за фронтом ударной волны, когда последняя входит в пузырьковую завесу. Рассмотрены также парогазовые пузырьки, совершающие малые радиальные колебания в жидкости под действием акустического поля. Показано, что капиллярные эффекты и фазовые переходы в совокупности приводят к новой резонансной частоте мелких паровых пузырьков, отличной от миннаэртовской. Получены выражения для частоты и декремента теплового затухания колебаний пузырьков. Определены эффективные коэффициенты теплообмена радиально пульсирующих пузырьков с жидкостью.